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MATHEMATICAL MODELING OF HEAT AND MASS TRANSFER IN FILM CONDENSATION

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UDC 536.244

The state of the problem of mathematical modeling of the heat and mass transfer during film condensation of a gas is considered.

Heat-exchange apparatus intended to transfer the substance from the gaseous to the liquid state — condensers — started to become widespread at the end of the 19th and beginning of the 20th centuries in connection with the appearance of steam-condensation turbines. At this time the thermal power of powerplant condensers is about 4 TW in the world. As a rule, the process of film condensation of vapor is realized in this apparatus.

The first theoretical work on heat exchange with condensation (W. Nusselt) appeared in 1916 [1], its results were the basis of methods of designing industrial condensators for decades. The majority of investigations on condensation reduced substantially to determining corrections to the Nusselt formulat to compute the heat-transfer coefficient. In 1954 Chernyi [2] published a paper which set the beginning of a qualitatively new stage in the development of a theory of condensation. The author of this paper first represented the model of film condensation in the form of two conjugate boundary layers (one of which is the condensate film), which permitted application of the well-developed apparatus of boundary-layer theory to investigate this process. This approach was utilized later to solve more complex condensation problems: vapor–gas systems [3, 4], a chemically reacting gas [5], etc.

Two categories of problems on film condensation exist: external and internal. For the external problems the gas stream parameters outside the boundary layer limits (temperature, velocity, composition in the case of multicomponent gas condensation) remain constant. In industrial apparatus with condensation on the outer surfaces, the film flow regime is kept mainly laminar or wave laminar.

The gas flow parameters vary continuously along the condensation surface during condensation inside tubes and channels, and the most diverse combinations of gas and liquid flow regimes can take place in the very same apparatus. In the general case, laminar film flow

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is initially replaced by wave and then by turbulent flow; because of the drop in its velocity at the end of the condensation section, the turbulent gas flow goes over into laminar.

Mathematical models to describe the film condensation process can provisionally be separated into four classes. In the first are solutions of the system of heat- and mass-transfer partial differential equations of parabolic type (boundary-layer equations) in an adjoint formulation. In the second, solutions of the adjoint system are more simple, for instance boundary-layer integral equations with empirical and other data relied upon to close the equations. The third class is of problems to approximate a boundary layer with suction when the equations are considered in the gas phase but the condensation process is modeled by suction through a solid wall. In the simplest models of the fourth class (which historically appeared first), transport is described only in the film while the action of the gas stream is taken into account by using appropriate boundary conditions on the interphasal surfaces.

For the general case of condensation of a chemically reacting gas, the equations for the gas phase have the following form

$$\rho w_x \frac{\partial w_x}{\partial x} + \rho w_y \frac{\partial w_x}{\partial y} = g(\rho - \rho_\infty)\sin\beta - \frac{\partial P}{\partial x} + \frac{1}{y^m}\frac{\partial}{\partial y}\left(y^m \mu_{\mathrm{eff}} \frac{\partial w_x}{\partial y}\right); \tag{1}$$

$$\frac{\partial}{\partial x}(\rho w_x) + \frac{1}{y^m}\frac{\partial}{\partial y}(y^m \rho w_y) = 0; \tag{2}$$

$$\rho w_x \frac{\partial H}{\partial x} + \rho w_y \frac{\partial H}{\partial y} = -\frac{1}{y^m}\frac{\partial}{\partial y}\left[y^m\left(q_{\mathrm{eff}} - \mu_{\mathrm{eff}} w_x \frac{\partial w_x}{\partial x}\right)\right]; \tag{3}$$

$$\rho w_x \frac{\partial C_k}{\partial x} + \rho w_y \frac{\partial C_k}{\partial y} = -\frac{1}{y^m}\frac{\partial}{\partial y}(y^m j_k) \pm I_k,\quad m = \begin{cases} 0, \\ 1. \end{cases} \tag{4}$$

The stream due to molecular transfer in the energy equation (3) takes account of the heat conductivity and enthalpy transfer by the diffusion streams:

$$q_{\mathrm{eff}} = -\lambda_f \frac{\partial T}{\partial y} + \sum_k h_k j_k. \tag{5}$$

The last term in the right side I_k in the convective diffusion equation (4) characterizes the source (sink) of the mass of the k-th component because of homogeneous chemical reactions.

In the general case, the system of equations for the liquid phase should be analogous to (1)-(4). However, as a rule the condensate can be considered as a one-component substance, and thereby the equation of convective diffusion (4) can be excluded from analysis. Moreover, it is shown in a number of papers [4-6, etc.] that in the case of condensation on the outer surfaces, the inertial forces and convective energy transfer to the condensate film can be neglected for working parameters actually encountered, i.e., the left side in (1) and (3).

The first term in the right side in the equation of motion (1) takes account of the angle between the condensation surface and the direction of gravity action. Turbulence is taken into account in a corresponding manner in the computation of the viscosity coefficient μ_{eff}.

The boundary conditions for the considered system of equations appear as follows for both phases: on the cooled surface (y = 0)

$$\begin{cases} w_{l\,x} = w_{l\,y} = 0, \\ w_{l\,x} = 0;\ w_{l\,y} = w_{l\,y}(x) \text{ — for film suction;} \end{cases} \tag{6}$$

$$\begin{cases} T = T_{\mathrm{w}}(x) \\ q = q_{\mathrm{w}}(x), \\ q = \alpha_0 (T_{\mathrm{w}} - T_0). \end{cases} \tag{7}$$

The conditions given for the external problem as y → ∞ are

$$T = T_\infty;\ w_x = w_{x\infty};\ C_k = C_{k\infty}. \tag{8}$$

For condensation within the pipes and channels, conditions are given at the input (x = 0):

$$w_x = w_x(y);\ \begin{cases} T = T(y), \\ h = h(y); \end{cases} C_k = C_k(y), \tag{9}$$

and after joining of the boundary layers ($y = y_w$) the condition of symmetry of the velocity, temperature, and concentration profiles is given:

$$\frac{\partial w_x}{\partial y} = \frac{\partial T}{\partial y} = \frac{\partial C_k}{\partial y}. \tag{10}$$

On the phase interface (y = δ) conditions of equality of the tangential velocity, temperature, and tangential stress components are given:

$$w_{lx} = w_x;\ T_l = T = T_s(C_k,\ P);\ \mu_l \frac{\partial w_{lx}}{\partial y} = \mu \frac{\partial w_x}{\partial y}, \tag{11}$$

energy flux

$$q_l = -\lambda_l \frac{\partial T_l}{\partial y} = q_{\text{eff}} + \Delta h_K g_\delta, \tag{12}$$

total convective mass flux

$$\rho_l \left(w_{lx} \frac{d\delta}{dx} - w_{ly} \right) = \rho \left(w_x \frac{d\delta}{dx} - w_y \right) = g_\delta, \tag{13}$$

mass flux of the k-th component

$$g_{lk} = C_k g_\delta + j_k. \tag{14}$$

G. G. Chernyi first derived the boundary conditions (11)-(13) from the general conservation laws for the case of pure vapor condensation [2]. The g_{lk} in (14) is the flux of the k-th component that goes over into the liquid phase because of dissolution, adsorption, etc. During condensation of liquid metals or strongly rarefied gases, the temperature of the liquid on the interphasal boundary $T_{l\delta}$ can differ noticeably from the gas temperature T_δ. The formulas to compute the temperature jump $\Delta T = T_\delta - T_{l\delta}$ are presented in [4, 7], for example.

In solving the problem in the approximation of a boundary layer with suction when the condensate film is not taken into account, there are naturally no boundary conditions on the interphasal surface and conditions are given for the gas phase on the wall (7). It should be noted that if a boundary condition of the first kind were given in this case, i.e., the wall temperature T_w, then this can only be the saturation temperature T_s during condensation of a pure vapor, while giving T_w for the condensation of a gas with noncondensable components automatically results in fixing the gas composition on the wall, which substantially distorts the actual pattern of the process.

Results of a numerical investigation of condensation by two-dimensional models in an adjoint formulation are presented in Figs. 1-3. The distribution of the tangential velocity component in a gas boundary layer with laminar condensation of the chemically reacting and the "frozen" gas with noncondensable components on the vertical surface is shown in Fig. 1 [5]. The nature of the velocity profiles indicates the substantial influence of natural convection on transfer in the gas phase. For comparison, curve 4 characterizes the profile w_x for the condensation of a pure saturated vapor. In this case the derivative $\partial w_x/\partial y$, and, therefore, the tangential stress on the phase interface also, reverse their sign.

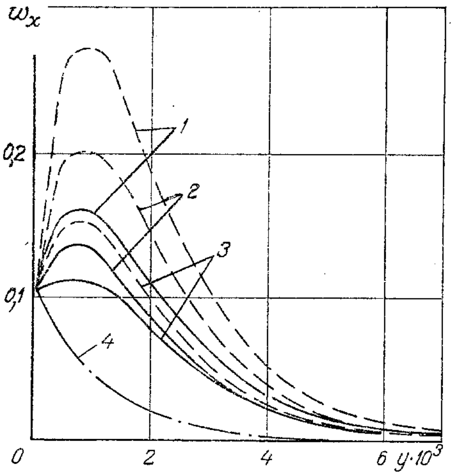

Fig. 1

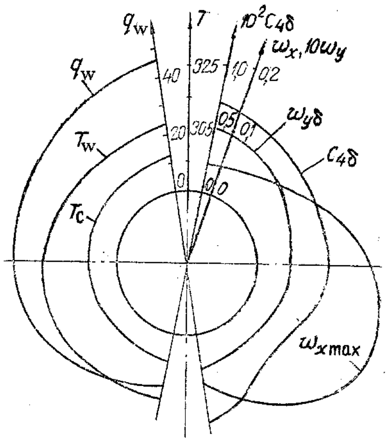

Fig. 2

Fig. 1. Tangential velocity component distribution in a gas boundary layer during condensation of the chemically reacting system $N_2O_4 \rightleftarrows 2NO_2 \rightleftarrows 2NO + O_2$ on a vertical surface: 1) $C_4 = 0.02$; 2) 0.01; 3) 0.005; 4) 0. Dashed line is the "frozen" case. w_x in m/sec, y in m.

Fig. 2. Change in the heat- and mass-transfer parameters along the perimeter during condensation of a N_2O_4 system on a horizontal pipe. q_w in kW/m²; T, °K; w_x, w_y in m/sec.

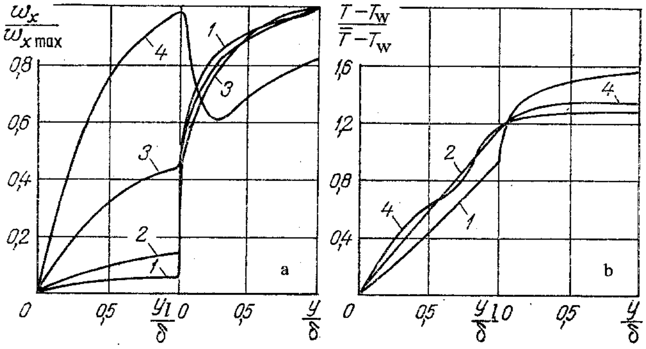

Fig. 3. Velocity (a) and temperature (b) profiles for condensation within vertical tubes: 1) x = 0.5 m; 2) 1.5; 3) 2.25; 4) 2.5 m.

The change in different heat- and mass-transfer parameters over the perimeter is shown in Fig. 2 for the condensation of a chemically reacting gas on a horizontal pipe [8]. The greatest heat fluxes in magnitude are realized in the upper part of a cylinder up to angles of 110-120° to the vertical. The maximal level of natural convection corresponds to approximately a 120° angle. Let us note that the normal velocity component in the gas boundary layer grows in absolute value during the passage from a flat vertical wall to a horizontal cylinder.

The change in the radial temperature and velocity profiles during condensation within a vertical pipe is shown in Fig. 3 [9]. The velocity profile at the input is given completely developed, and the temperature is homogeneous. The radial velocity profiles retain their form although the continuously dropping gas velocity exceeds the continuously growing fluid velocity. Starting with a certain time, the condensate film already entrains the gas stream. Near this zone retuning of the velocity profile occurs, the tangential stress diminishes to zero on the phase interface and then reverses its sign, while an inflection point appears in the velocity distribution over the gas.

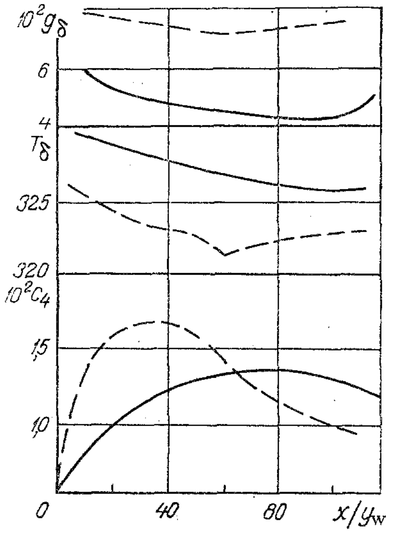

Fig. 4

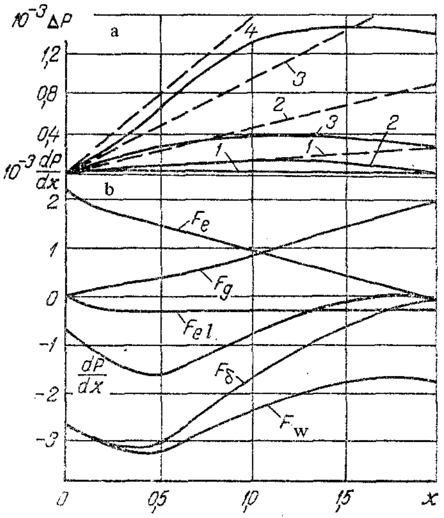

Fig. 5

Fig. 4. Influence of the condensate film on the change of the heat- and mass-transfer parameters over the length during condensation of a chemically reacting gas in a pipe. Solid lines are with the film taken into account, and the dashes with complete suction of the film; g_δ, kg/m^2·sec; T_δ, K.

Fig. 5. Change in static pressure (a) for different gas velocities at the entrance [1) w_{in} = 2.5 m/sec; 2) 5; 3) 7.5; 4) 10 m/sec], in pressure gradient and its component for w_{in} = 10 m/sec (b) with length. Dashed lines correspond to gas flow without condensation; ΔP, dP/dx, Pa; x in m.

Temperature profiles are shown for the case when the initial enthalpy of the heating is approximately 15% of the heat of condensation. Hence, the gas temperature profile acquires the characteristic curvature after the thermal stabilization section. However, the condensation process is accompanied by an elimination of heating from the gas phase, and at the end of the pipe the gas temperature over the whole section is practically equal to the condensation temperature. While the flow regime remains laminar or transitional in the condensate film, the temperature distribution is almost linear.

A comparison of the results for computing the condensation of a chemically reacting gas in a pipe in an adjoint formulation and in the approximation of the theory of a boundary layer with suction is shown in Fig. 4 for boundary conditions of the third kind on the wall. The nature of the change in the mass- and heat-transfer parameters over the length differs noticeably for these variants. Let us note that the problem with film suction through the solid wall acquires independent interest in recent times since the heat-transfer intensity grows substantially in this case [10].

The results obtained by using two-dimensional mathematical models permit the investigation of the mass- and heat-transfer mechanism during condensation, but they are difficult to use in engineering practice; moreover, they require significant machine time expenditures. One-dimensional models based on the numerical integration of first-order differential equations describing the change with length in the parameters of each of the phases separately averaged over the cross section turn out to be quite convenient for practical computations on a computer. The strict derivation of one-dimensional equations from the system of initial differential equations (1)-(4) is elucidated in [11]. In particular, expressions to compute the pressure gradient during condensation in a pipe have the following form for both phases

$$\frac{dP_l}{dx} = F_g + F_w + F_\delta + F_{el}; \tag{15}$$

$$\frac{dP}{dx} = F_\delta + F_e, \tag{16}$$

where $F_g = (\rho_l - \rho)g$ is the gravity force, $F_w = -\frac{\Pi_w}{S_l}\tau_w$ is the wall friction, $F_\delta = -\frac{\Pi_\delta}{S_\delta}\tau_\delta + \frac{w_{x\delta}}{S_\delta}\frac{dG}{dx}$ is the effective friction on the phase interface with the transverse mass flux taken into account, $F_e = \frac{2\pi}{S_\delta}\frac{d}{dx}\int_{S_\delta}\rho w_x^2 y^m dy$ is the change in the kinetic energy of the gas flux, and $F_{el} = \frac{2\pi}{S_l}\frac{d}{dx}\int_{S_l}\rho_l w_{lx}^2 y^m dy$ is the change in the kinetic energy of the fluid flow.

The change in total pressure along the length, as computed from (15) and (16), is represented in Fig. 5 for different gas velocities at the entrance w_{in}, the pressure gradient and its components for w_{in} = 10 m/sec. Let us note firstly that a domain with positive static pressure gradient is always observed in the final section of the pipe in the versions under consideration. The "positive" contribution to the pressure gradient is induced by the gravity force and the diminution in the kinetic energy of the gas flux, and the "negative" contribution is from friction on the wall and on the phase interface. As a result of the algebraic addition of these factors, the pressure drop in the gas flux being condensed is always less than in a single-phase flow, other conditions being equal.

The results presented indicate that mathematical modeling at this time is an effective method of studying heat- and mass-transfer processes for film condensation.

NOTATION

C_k, relative mass concentration of the k-th component; G, flow rate; g, acceleration of gravity; $H = h + w_x^2/2$, total stream enthalpy; h, enthalpy; h_k, enthalpy of the k-th component; Δh_k, specific heat of condensation; j_k, diffusion flux of the k-th component; P, pressure; T, temperature; T_s, saturation temperature; S, cross-sectional area; q, specific heat flux; w_x, w_y, velocity components; x, y, coordinates; δ, condensate film thickness; μ, viscosity; Π, perimeter; ρ, density; and τ, tangential stress. Subscripts: eff, effective; f, "frozen"; in, input; w, wall; *l*, liquid; c, cooling medium; δ, phase interface; 4, O_2 component in the system $N_2O_4 \rightleftarrows 2NO_2 \rightleftarrows 2NO + O_2$.

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MOTION OF AN ELASTOVISCOUS LIQUID WITHIN A TUBE AFTER REMOVAL OF A PRESSURE DIFFERENTIAL

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UDC 532.135

The nonisothermal flow of a nonlinear hereditary liquid within a ring-shaped channel after instantaneous removal of a pressure differential is studied.

The present study is a continuation of [1], which considered flow development after impulsive application of a pressure differential. We will now consider halting of a flow upon instantaneous removal of a pressure differential. Analytical solutions of this problem for the linear formulation have been presented in [2, 3]. In [4] a numerical calculation of flow in a circular tube for a two-constant Oldroyd model was performed. However, until the present there has been no study of the effect on rheodynamics of such important factors as nonisothermal conditions, the relaxation time spectrum, and the dependence of that spectrum and the relaxation moduli on shear velocity.

As in [1], we will employ a nonlinear integral rheological equation of state (RES):

$$\mathbf{T} = \int_{-\infty}^{t} m[t-t',\ S_{\mathbf{D}}(t')]\left[\left(1+\frac{\varepsilon}{2}\right)(\mathbf{C}_t^{-1}(t')-\mathbf{E}) + \frac{\varepsilon}{2}(\mathbf{C}_t(t')-\mathbf{E})\right]dt',$$

$$m = \sum_{k=1}^{\infty} \frac{\eta_k}{\lambda_k} f_k(S_{\mathbf{D}}(t')) \exp\left[-\int_{t'}^{t} \frac{g_k(S_{\mathbf{D}}(t''))}{\lambda_k}\,dt''\right], \quad (1)$$

$$S_{\mathbf{D}}^2 = 2\,\mathrm{tr}\,\mathbf{D};\ \eta_k = \eta_0/\zeta(\alpha)\,k^{\alpha};\ \lambda_k = \lambda/k^{\alpha},\ 1.5 \lesssim \alpha \lesssim 8.$$

Calculations were performed for a liquid the properties of which are independent of the deformation rate $f_h = g_h = 1$, and for three nonlinear models — the Bird–Carro (BC), Meister (M), and Macdonald–Bird–Carro (MBC) (see [1, 5]). The problem is formulated mathematically for a tube, the length of which significantly exceeds the extent of the hydrodynamic and thermal input segments:

$$\rho \frac{\partial v_z}{\partial t} = -\frac{\partial p}{\partial z}(t) + \frac{1}{r}\frac{\partial}{\partial r}(rT_{rz}),\ R_1 \leqslant r \leqslant R_2, \quad (2)$$

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